Scaling in drop distributions: An application in combustion

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Scaling in the distribution of fragments in the explosion of a fuel drop is demonstrated. Consequences for the process of emulsion combustion are discussed. A relation between unburned matter and the value of the scale exponent of the distribution was obtained.

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I. INTRODUCTION

Water-fuel emulsions are widely recognized as a good option for combustion improvement. In this case, the process of fuel burning becomes more efficient, leading to soot reduction and net fuel profit [1].

One of the reasons why emulsions improve combustion processes is that, as observed by Ivanov and Nefedov [2], emulsified fuel droplets tend to explode when heated, hereby solving the problem of fuel atomization. The emulsified fuel drop is a complex medium formed by fuel, water, and surfactants. In conditions of high temperature, such as those existing in the combustion chamber, a sudden process of bubble nucleation, leading to the splitting of the original drop in some fragments, is favored. This process can still happen in some of the occurring fragments leading to subsequent fractioning of the system. The resultant distribution of secondary droplets presents a larger effective surface to the combustion than that of the original drop. Experimental studies of the combustion of emulsion droplets confirmed their disruptive behavior during the combustion [3]. It is interesting to evaluate this combustion improvement of the emulsified drop with respect to the combustion of pure fuel.

As a first step it is necessary to describe the distribution of fragments produced by the drop burst. The key problem is to find the law of fragment distribution by size. Section II is devoted to this problem, where demonstrative experiments of fuel drop breaking show that the size distribution of fragments obeys a scaling law. This fact allows us to establish a relationship between the scale exponent of the distribution and the quantity of unburned matter in one of the combustion steps. We show in Sec. III that it is possible to speak about an "ideal" or "complete" combustion condition for a given value of the scale exponent. In Sec. IV we give an interpretation of this complete combustion in terms of the self-similarity of the distribution.

II. DEMONSTRATIVE EXPERIMENTS

Scaling has proved to be a widely observed phenomenon in which the cumulative number per unit volume $N(r)$ of members in some collection whose sizes are larger than r bears a power-law form

$$N(r) \sim r^{-x} . \tag{1}$$

Here x is the so-called scaling exponent. As examples of this law, we can mention the Korčak law for cumulative number of islands, the distribution of lunar craters, atmospheric aerosols, etc. These and other examples are discussed in [4], presenting also a simple geometric model to describe the fracture process.

As was pointed out in the Introduction, emulsion drops experience a fragmentation process in an early stage of combustion and it is valid to pose the problem about the validity of (1) to describe the distribution of fragments. In order to do this we simulated experimentally the process of drop microexplosions as follows (see Fig. 1).

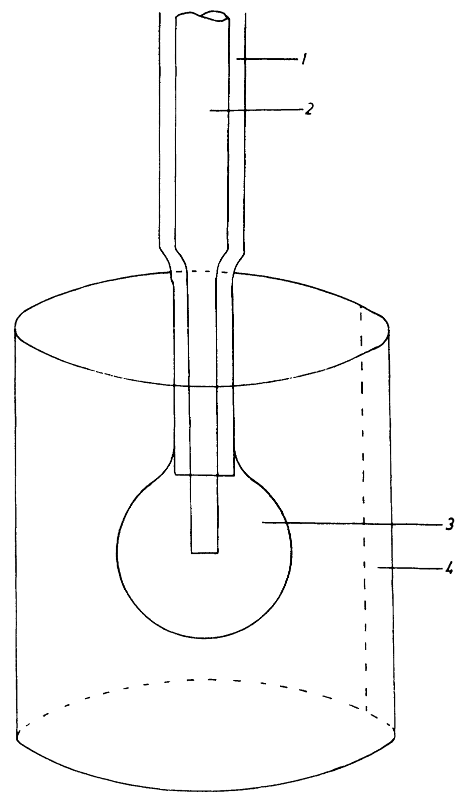

FIG. 1. Experimental setup for demonstration of scaling distribution of fragments produced by the burst of a fuel droplet: (i) capillary in which the drop is hanging; (ii) thin capillary through which air is injected into the interior of the drop; (iii) drop of fuel oil; (iv) transparent sheet of acetate, collector of the fragments produced by the fractioning of the drop.

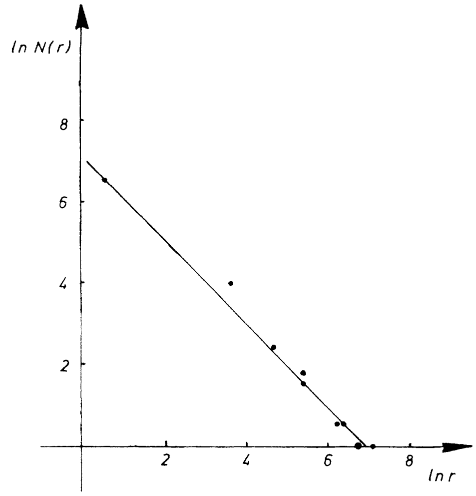

FIG. 2. Logarithmic plot of the cumulative number of fragments vs radius for the experiment shown in Fig. 1, showing scaling behavior. In this graph $x \approx 1.5$. Units of r are arbitrarily measured in the scale of the microscope.

In a glass capillary 1 there is a hanging drop of fuel oil 3. An even thinner capillary 2 goes throughout the capillary 1 and enters the drop, in order to act as some kind of "bubble generator." When air is injected by 2 a bubble is formed inside the drop. The injection process is strong enough to burst the drop, thus mimicking the process of fragmentation for emulsified fuel drops. Fragments were collected on a rolled transparent plate of acetate 4 and counted with the help of a microscope. All carried out experiments were performed under the same conditions and with the same sample of fuel oil. All fragments within the resolution of the microscope which reached the plate were measured. The results are shown in Fig. 2, where the logarithm of the cumulative number of fragments is plotted as a function of the logarithm of their radius. A straight line was always obtained in this dependence. The fragments of the original drop, when impinging on the plate, stay there and, because of the characteristics of the fuel oil (surface tension, viscosity), are only slightly deformed from their spherical shape, so the spherical shape of the collected fragments and their dark color simplifies to some extent the process of counting and measurement.

The process of measurement and counting of fragments gives similar results for arbitrarily chosen sectors of the plate. A scaling law with similar values of the critical exponent was always obtained. The experiments described above clearly show that in the fracture of drops, fragments are distributed according to a power law. Though fracture processes have been studied and modeled (e.g., in [5]), there are no reports on this kind of experiment in "liquid drop fracture" revealing scaling also in this case.

III. CONSEQUENCES FOR COMBUSTION

Clearly, the process of combustion is improved if the original drop breaks up into many pieces because of the increase of fuel-air interface. Once we know the distribution law of fragments, it is not difficult to evaluate the consequences of this distribution for the combustion of the oil drop. Following Eq. (1) we may put

$$N(r)=ar^{-x} \tag{2}$$

for the cumulative number of fragments. In this case, the number of fragments (or secondary droplets) with radius between r and $r+dr$ is, taking the derivative of (2) with respect to r,

$$n(r)=axr^{-x-1} . \tag{3}$$

We may consider that the initial drop volume V can be expressed through the volume of the largest fragment of radius R, i.e.,

$$V=\tfrac{4}{3}\pi\beta R^3 , \tag{4}$$

where $\beta>1$ is a constant. The distribution function can be normalized by

$$ax\int_0^R r^{2-x}dr=\beta R^3 , \tag{5}$$

which expresses that the sum of volumes of the fragments produced by the explosion of the drop is equal to the volume of the original one. This condition requires that the fragmentation process is fast enough so fuel consumption can be neglected during that time. Equation (5) gives

$$a=\frac{3-x}{x}\beta R^x . \tag{6}$$

Finally,

$$n(r)=(3-x)\beta R^x r^{-x-1} \tag{7}$$

is the number of fragments with radius between r and $r+dr$. We now take into account this distribution in the evaluation of the unburned volume fraction of the liquid mass that was not gasified in the combustion chamber and consequently not burned in the combustion process. Therefore, information about the efficiency of the process can be obtained by evaluating the amount of unconsumed fraction in the set of fuel droplets.

In order to simplify the calculations we will suppose that, after the fragmentation process has been accomplished, each secondary droplet burns independently of the others, so that each drop can be considered as being surrounded by an infinite quantity of gaseous oxidizer. In [6], more complicated situations are considered and could be used to make a less idealistic model for combustion of drop distribution, though essential results would not be different from those obtained here.

As is known, the combustion of isolated liquid drop has three main step or phases.

(a) The ignition delay time, during which surface evaporation raises the concentration of the fuel vapor to the point where spontaneous ignition occurs. During this period, the diameter of the drop is negligibly changed.

(b) The burning phase, during which the droplet size decreases until all the liquid fuel is evaporated.

(c) The postdroplet period during which the residual fuel is consumed.

We will suppose that the first step is fast and we center our attention in the consequences of the scaled distribution of droplets in the second step. In a simple model the radius of the droplet varies with time according to the law [7]

$$r^2=r_i^2-kt \ . \tag{8}$$

The constant k is called *the characteristic evaporation constant of the fuel.* r_i is the initial drop radius and r is the radius at time t.

Now let us introduce the "critical radius" r_0 as that for which the drop is just consumed for a time τ (in our simple model, τ being determined essentially by the stay time in the combustion chamber); the final radius, once that time has elapsed, can be found by applying (8). If the initial radius has the value r_0, the final radius will be zero after τ. Fragments with size less than that determined by r_0 will be completely consumed, whereas those with size larger than that determined by r_0 will contribute to the quantity of unburned matter. Hence, each drop with initial radius r larger than r_0 will give a volume of unconsumed fuel given by $\frac{4}{3}\pi(r^2-k\tau)^{3/2}$, but by definition $r_0=\sqrt{k\tau}$, so the quantity of unconsumed fuel given by that drop is

$$i=\tfrac{4}{3}\pi(r^2-r_0^2)^{3/2} \ . \tag{9}$$

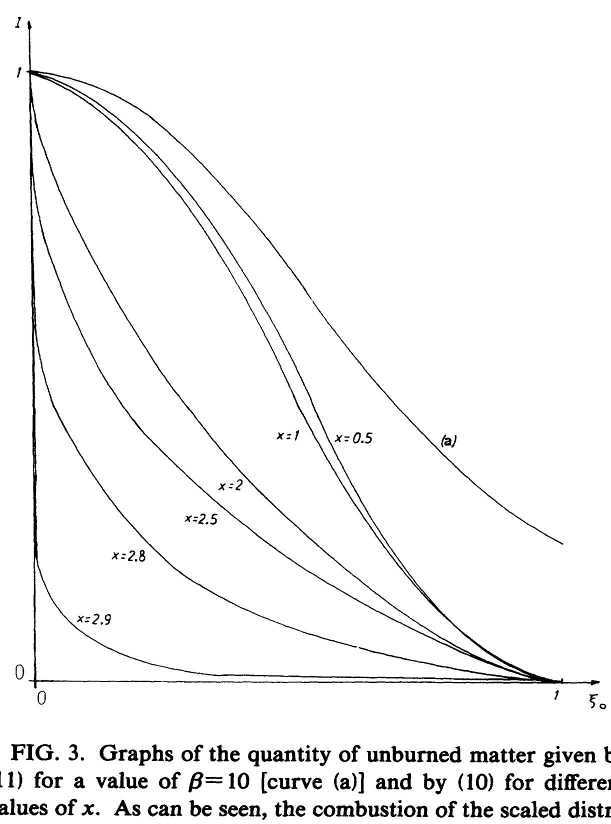

FIG. 3. Graphs of the quantity of unburned matter given by (11) for a value of $\beta=10$ [curve (a)] and by (10) for different values of x. As can be seen, the combustion of the scaled distribution of fragments leads to a sharp improvement of combustion. As $x\rightarrow 3$ the condition for complete consumption is reached.

With (7) and (4) the quantity of unburned matter can be calculated; if (9) is the quantity of unburned liquid fuel given by a drop of initial radius r, by summing the contributions of the $n(r)$ drops from the critical radius r_0 up to the largest radius of the fragments R, finally gives

$$I=(3-x)\int_{\xi_0}^{1}(\xi^2-\xi_0^2)^{3/2}\xi^{-x-1}d\xi \ . \tag{10}$$

Here I is the quantity of unburned liquid fuel given in units of the volume of the original (mother) drop; $\xi=r/R$, $\xi_0=r_0/R$. On the other hand, the value of the unburned volume fraction for a "nonfractioning" drop is (expressed in units of the drop mass)

$$I_0=\left[1-\frac{\xi_0^2}{\beta^{2/3}}\right]^{3/2} \ . \tag{11}$$

In Fig. 3 we have plotted the results of (10) for different values of x and of (11) as a function of $\xi_0=\sqrt{k\tau}/R$. From the figure it can be seen that the process of consumption is always improved with "exploding" or "fractioning" drops, and becomes even faster as x approaches 3. In this case, the consumption appears to be instantaneous for all the distribution of fragments, reaching some kind of complete combustion in this step of the process.

IV. DISCUSSION OF THE RESULTS

The curious fact that consumption of the distribution is instantaneous when x reaches the value 3 can be interpreted in terms of the self-similarity of the distribution, following the same analysis as in [4].

For this, we may analyze a cubic volume with sides of unit length. The cumulative number of drops in this cube is given by (1). Next, select a piece of this cube with side λ^{-1} ($\lambda>1$). The cumulative number for this small volume is

$$N_{\lambda^{-1}}(r)\sim\lambda^{-3}r^{-x} \ . \tag{12}$$

If we now look at this volume with a microscope of magnification λ, the piece is transformed in the original cube, the resolution is increased in this factor, and the new limit for resolution is given by $\lambda^{-1}r$. A new distribution is observed with cumulative number

$$N_{\lambda^{-1}}(\lambda^{-1}r)\sim\lambda^{x-3}r^{-x} \ . \tag{13}$$

Hence it is easy to see that the condition $x=3$ implies invariance of the cumulative number when the scale is changed, i.e., the condition of self-similarity of the distribution. There are many ways to see that this value is impossible to reach (x cannot be larger than the dimension of the space). Application of the geometrical model of fracture proposed in [4] leads to the results that x is always less than the value of the dimension of the space. Therefore, in this model, the condition for "ideal combustion" of fuel (understood as the condition for fastest consumption of the distribution of drops) is equivalent to scale invariance.

Finally, we want to point out that, though we have analyzed the implications of scaling in combustion, this phenomenon has consequences in all phenomena involving distribution of particles and transport throughout interfaces.

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